Computer-assisted Proofs of Non-Reachability for Linear Parabolic Control Problems with Bounded Constraints

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Austrian Numerical Analysis Day

May 8th, 2026

Université Paris Cité

LABORATOIRE MAP5

ALPEN-ADRIA UNIVERSITÄT KLAGENFURT

Summary

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Control system

Consider the system:

$$\begin{cases} \dot{y}(t) + Ay(t) = Bu(t) & \forall t \in [0, T] \\ y(0) = y_0 \in X \\ u(t) \in \mathcal{U} \subset U & \forall t \in [0, T]. \end{cases} \tag{S}$$

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or:

$$\forall t \in [0,T], \quad y(t; y_0, u) = e^{-tA} y_0 + L_t u.$$

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Denote as well the constraint set:

$$E_{\mathcal{U}} = \{u \in L^2(0,T;U), \quad \forall t \in [0,T], \ u(t) \in \mathcal{U}\}.$$

Reachability

Definition

A target set $\mathcal{Y}_f$ is $\mathcal{U}$-reachable from y_0 in time T if :

$$\exists u \in E_{\mathcal{U}}, \exists y_f \in \mathcal{Y}_f, \quad y(T; y_0, u) = y_f.$$

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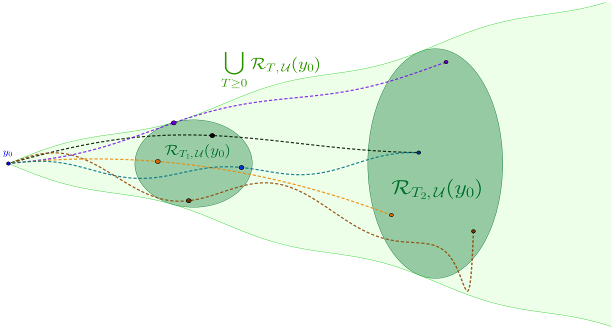
Definition

A target set $\mathcal{Y}_f$ is $\mathcal{U}$-reachable from y_0 in time T if :

$$\mathcal{R}_{T,\mathcal{U}} \cap \mathcal{Y}_f \neq \emptyset,$$

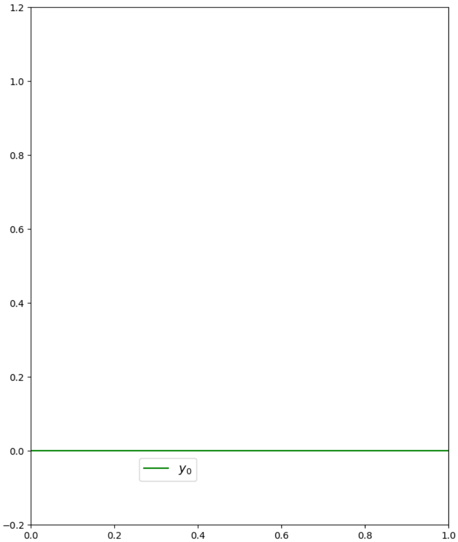
where $\mathcal{R}_{T,\mathcal{U}}(y_0) = e^{-TA} y_0 + L_T E_{\mathcal{U}}$ is the set of $\mathcal{U}$-reachable targets (from y_0 in time T), called reachable set.



Motivation

$$\forall\, t,x \in [0,T]\times[0,1],$$

$$\begin{cases} \dot{y}(t,x) - \Delta y(t,x) = 0 \\ y(0,x) = y_0(x) = 0 \\ y(t,0) = y(t,1) = 0. \end{cases}$$



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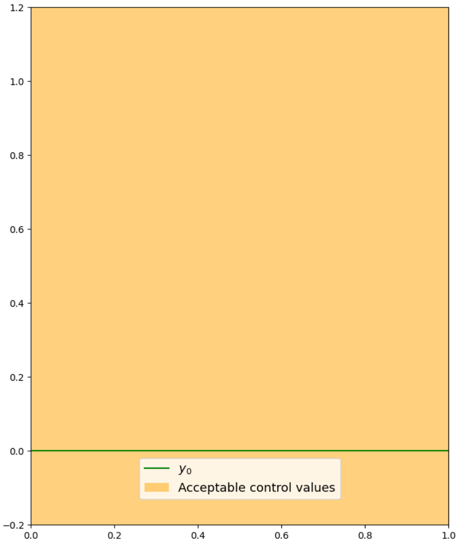

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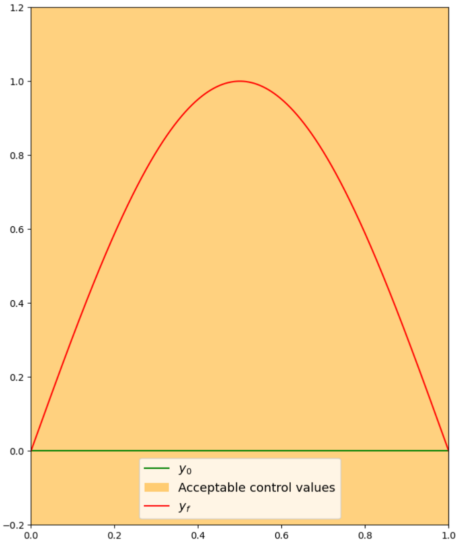
Tools literature can give you:

- Approximate controllability for any $T > 0$
- Spectral methods: explicit appropriate controls



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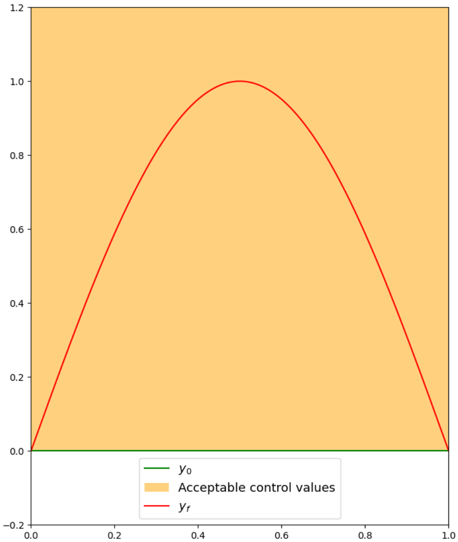
Tools literature can give you:

- Approximate controllability for positive targets, for $T > 0$ large enough [LTZ17]
- Spectral methods: explicit appropriate controls (sometimes)



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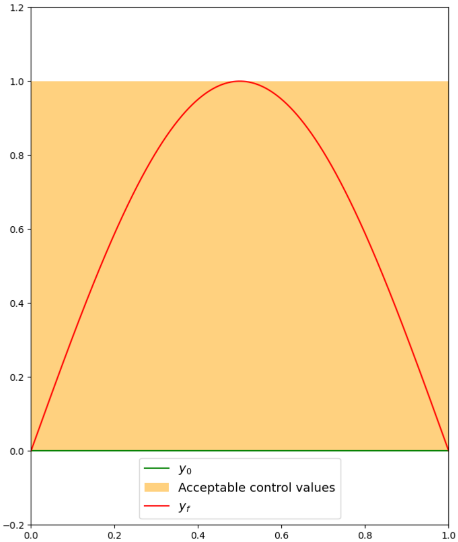
Tools literature can give you:

- Regularity criterion for non-reachability [CR22], comparison principle
- Spectral methods: explicit appropriate controls (if you're lucky)



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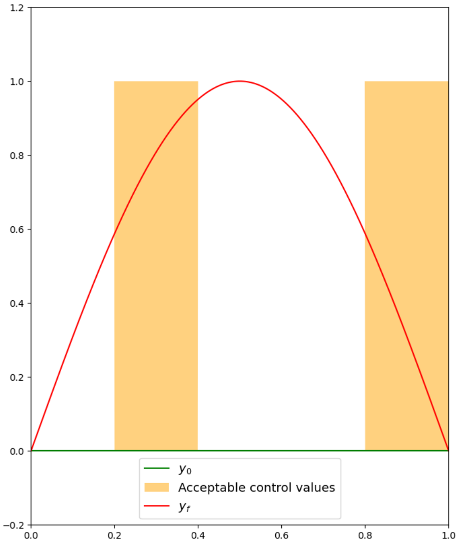
$\forall\, t,x \in [0,T] \times [0,1],$

$$\begin{cases} \dot{y}(t,x) - \Delta y(t,x) = \mathbb{1}_\omega u(t,x) \\ y(0,x) = y_0(x) = 0 \\ y(t,0) = y(t,1) = 0 \\ 0 \leq u(t,x) \leq 1 \\ y(T,x) = y_f(x) = \sin(\pi x). \end{cases}$$



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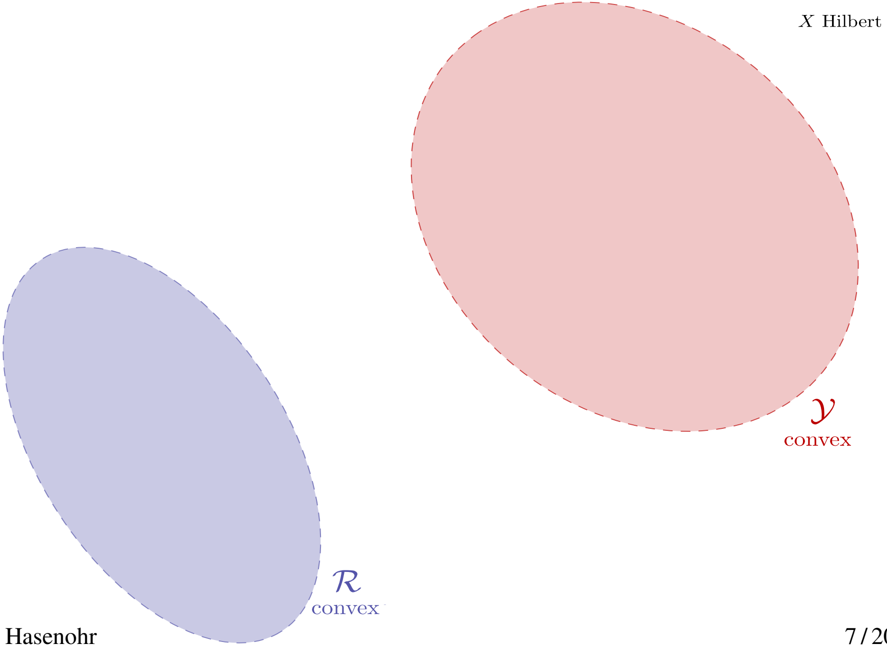
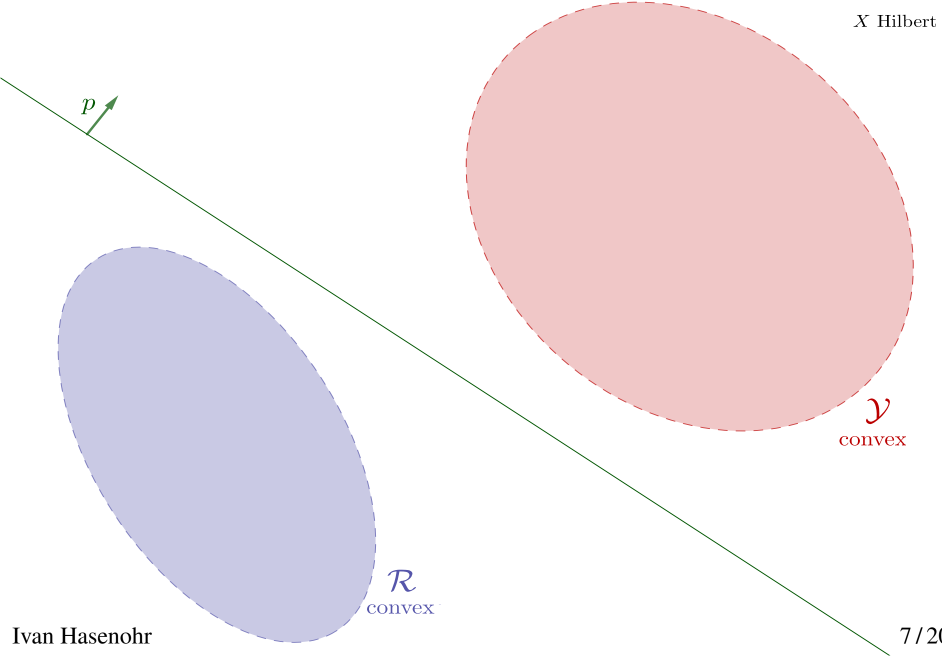
- Regularity criterion for non-reachability [CR22], comparison principle
- Numerical insights: no proofs

Summary


X Hilbert
$\mathcal{Y}$
convex
$\mathcal{R}$
convex



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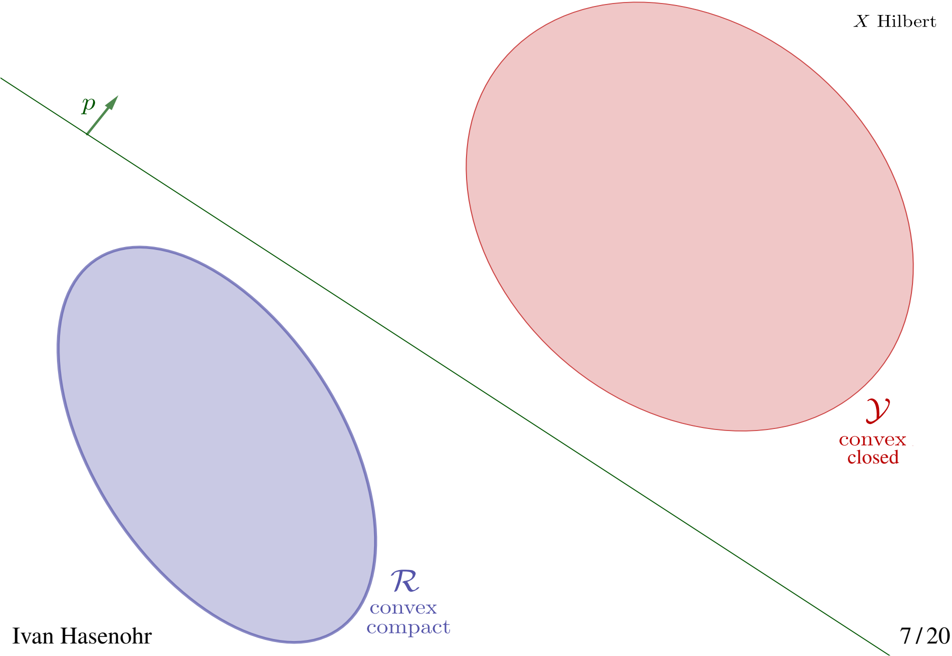
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p
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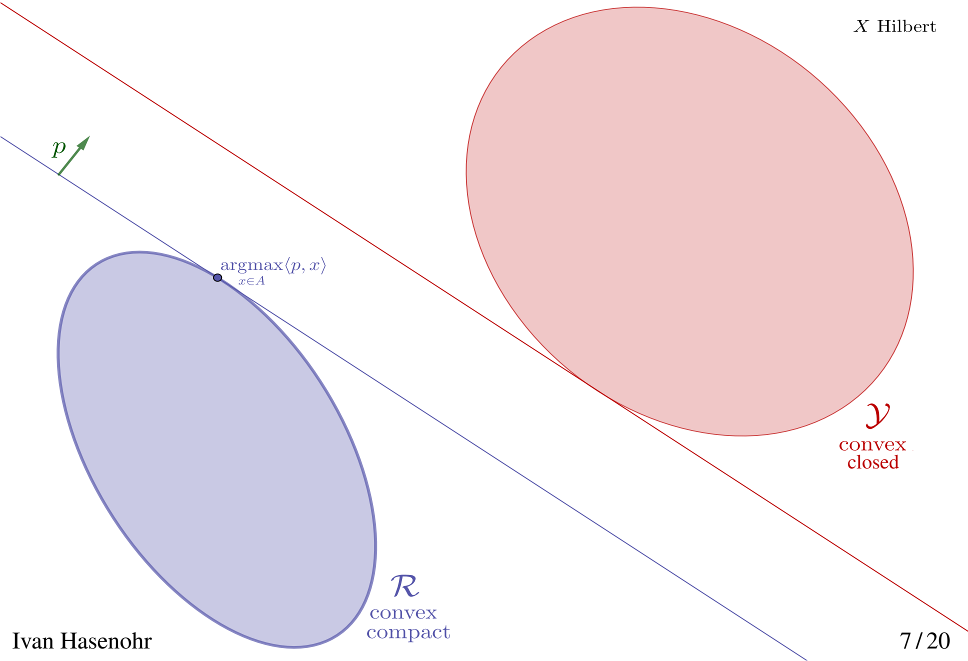
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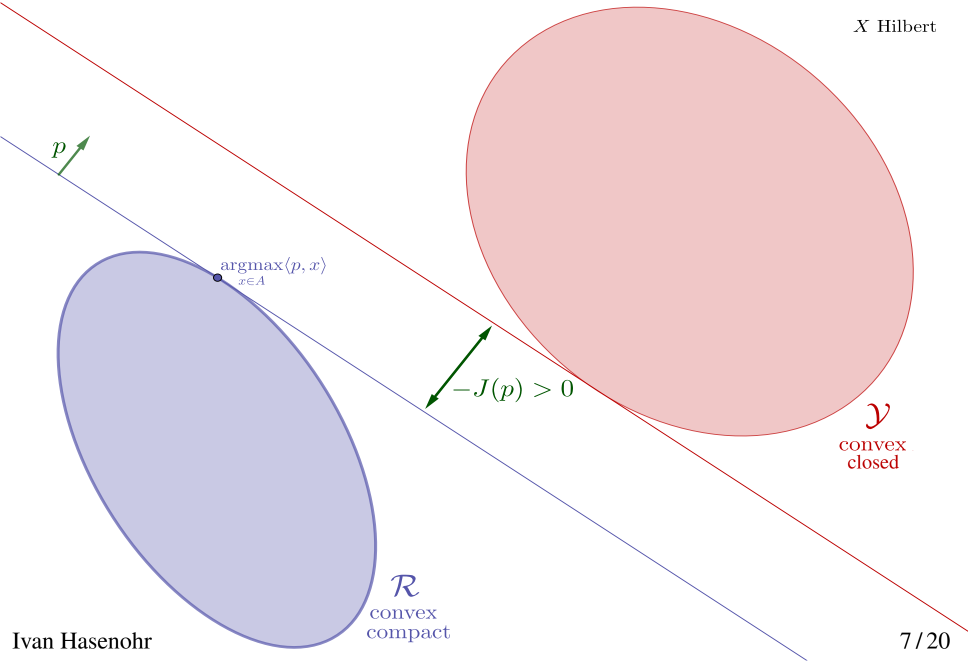
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Separation functional

Let us denote

$$J : p \mapsto \sigma_{\mathcal{R}}(p) + \sigma_{\mathcal{Y}}(-p),$$

where

$$\sigma_{\mathcal{R}} : p \mapsto \sup_{x \in \mathcal{R}} \langle p, x \rangle \quad \text{et} \quad \sigma_{\mathcal{Y}} : p \mapsto \sup_{x \in \mathcal{Y}} \langle p, x \rangle.$$

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Theorem

Assume $\mathcal{R}$ convex and compact, $\mathcal{Y}$ convex and closed. It follows

$$\exists p \in X, \quad J(p) < 0 \qquad \Longleftrightarrow \qquad \mathcal{R} \cap \mathcal{Y} = \emptyset.$$

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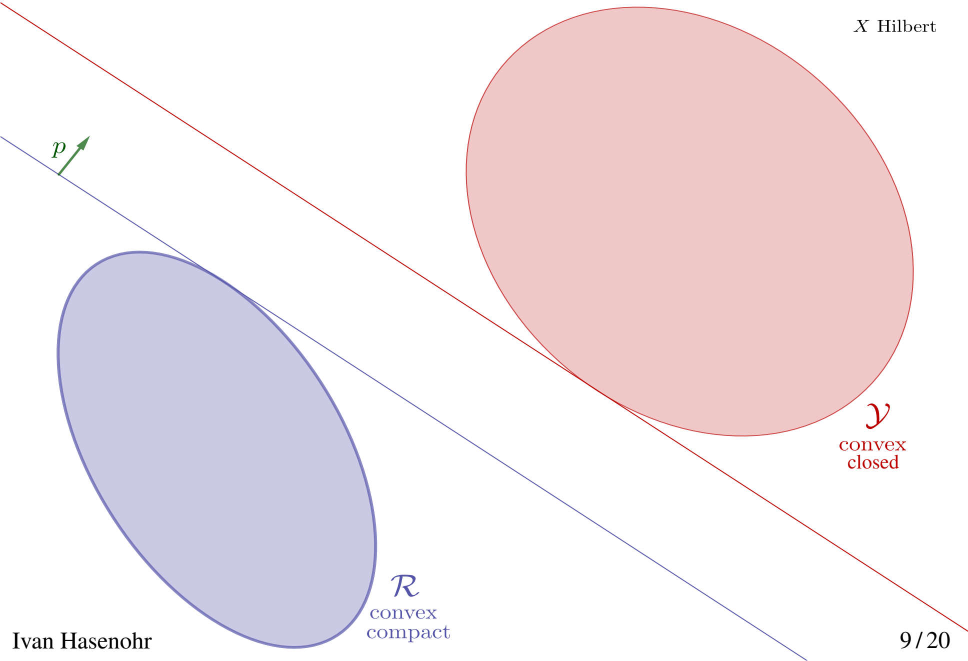
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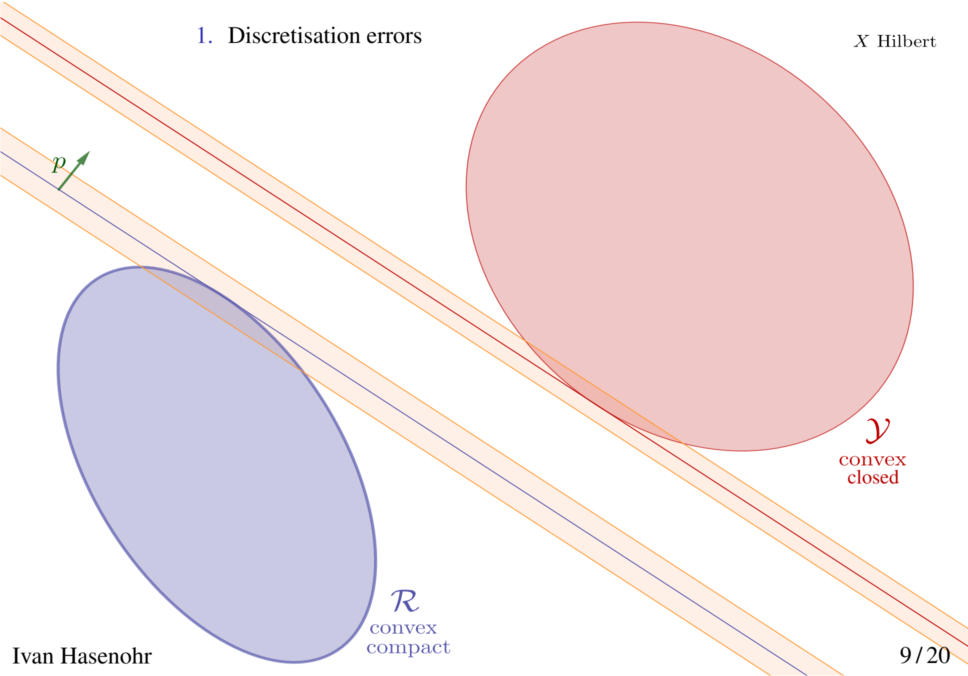

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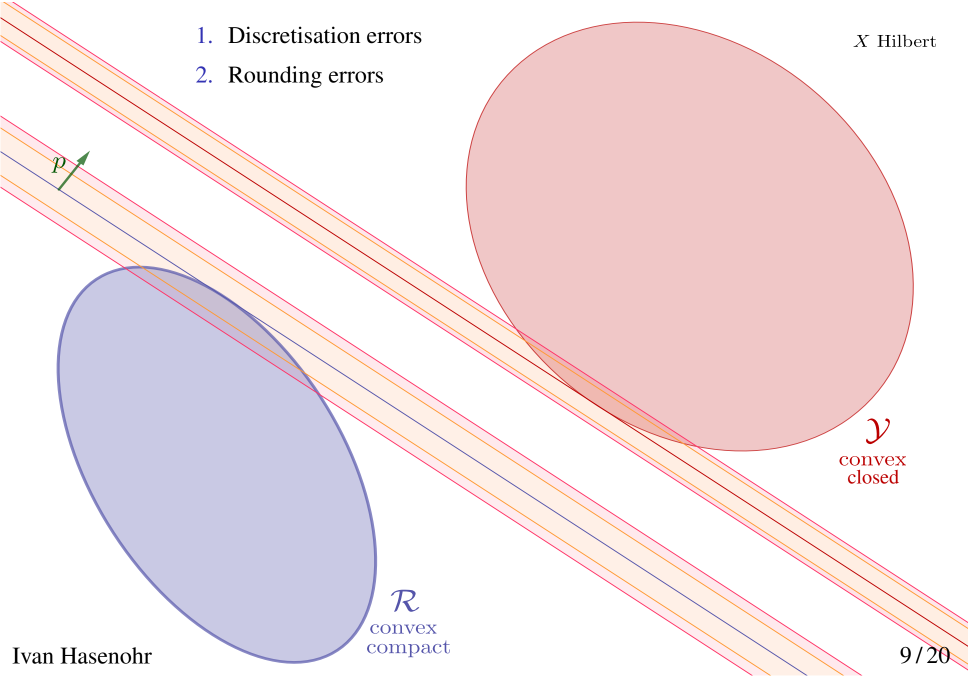
2. Rounding errors



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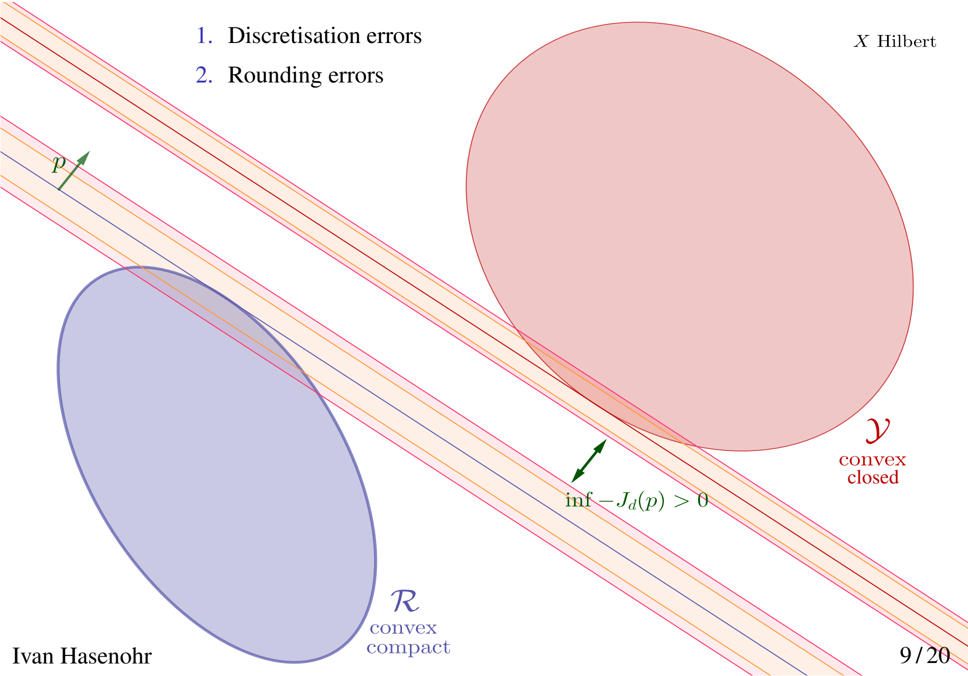

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6. Check that $J_d(p_{fh}) + e_r(p_{fh}) + e_a(p_f) < 0$, and thus that $J(p_f) < 0$ and

$$\mathcal{R} \cap \mathcal{Y} = \emptyset.$$

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Reachability

Recall: $\mathcal{Y}_f$ is $\mathcal{U}$-reachable from y_0 in time T if and only if

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If $\mathcal{U}$ is non-empty, convex, closed, bounded, then $\mathcal{R}_{T,\mathcal{U}}(y_0)$ is convex and weakly compact.

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If $\mathcal{U}$ is non-empty, convex, closed, bounded, and if $\mathcal{Y}_f$ convex and closed, denoting

$$J(p_f) = \sigma_{\mathcal{R}_{T,\mathcal{U}}(y_0)}(p_f) + \sigma_{\mathcal{Y}_f}(-p_f),$$

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Framework

Recall the control system:

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- $A : V \to V'$ such that: $\exists 0 < a_0 \leq a_1$,

$$\forall v, w \in V \times V, \quad \begin{cases} |\langle Av, w\rangle| & \leq \; a_1 \|v\|_V \|w\|_V, \\ \mathrm{Re}\langle Av, v\rangle & \geq \; a_0 \|v\|_V^2. \end{cases}$$

Discretisation

We discretise

$$J(p_f) = \int_0^T \sigma_{B\mathcal{U}}(e^{tA^*}p_f)\,\mathrm{d}t + \langle p_f, e^{-TA}y_0\rangle + \sigma_{\mathcal{Y}_f}(-p_f),$$

over a finite-dimensional subspace $V_h \subset V$ and a regular time grid of parameter $\Delta t = \frac{T}{N_t}$ using a implicit Euler scheme: $\forall p_{fh} \in V_h$,

$$J_{\Delta t,h}(p_{fh}) = \Delta t \sum_{n=1}^{N_t} \sigma_{B\mathcal{U}}((\mathrm{Id} - \Delta t A_h^*)^{-n} p_{fh}) + \sigma_{\mathcal{Y}_f}(p_{fh}) + \left\langle (\mathrm{Id} - \Delta t A_h^*)^{-N_t} p_{fh}, y_0 \right\rangle.$$

Discretisation error

Theorem (I.H., C. Pouchol, Y. Privat, C. Zhang)

Under the additional a-priori assumption on V_h

$$\forall f \in X, \quad \inf_{v_h \in V_h} \|A^{-1} f - v_h\|_V + \inf_{v_h \in V_h} \|(A^*)^{-1} f - v_h\|_V \leq C_0\, h \|f\|,$$

we have

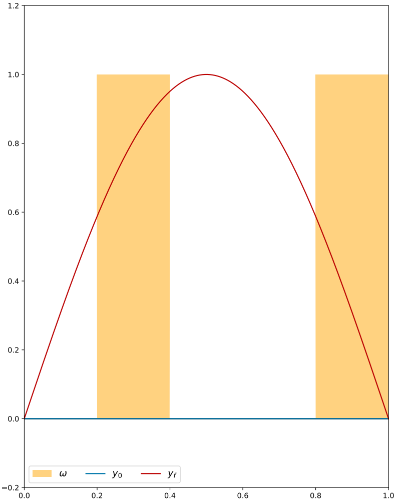
$$\begin{aligned} |J(p_f) - J_{\Delta t, h}(p_{fh})| \leq \quad & \tfrac{1}{2} MT \|B\| \, \Delta t \|A^* p_f\| \\ & + (\|y_0\| + MT\|B\|) \left(C_1 h^2 + C_2 \, \Delta t \right) \|A^* p_f\| \\ & + \left((\|y_0\| + MT\|B\|) C_3 + \|\mathcal{Y}_f\| \right) \|p_f - p_{fh}\|. \end{aligned}$$

Summary

Heat equation

$\forall t, x \in [0, T] \times [0, 1]$,

$$\begin{cases} \dot{y}(t,x) = \Delta y(t,x) + \mathbb{1}_\omega u(t,x) \\ y(0,x) = y_0(x) = 0 \\ y(t,0) = y(t,1) = 0 \\ 0 \leq u(t,x) \leq 1 \\ y(T,x) = y_f(x) = \sin(\pi x). \end{cases}$$



Spatial discretisation

Here we have:

- $X = L^2(0,1)$ the state space
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For the choice of V_h

1. $V_h \subset \mathcal{D}(A^*)$ (cubic splines, spectral methods. . .) :
   - Advantages : no interpolation needed, less discretisation errors
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     $\Longrightarrow$ easy for $\mathbb{P}_1$ using cubic splines.

Non-reachable states for the heat equation

$\forall t, x \in [0, T] \times [0, 1]$

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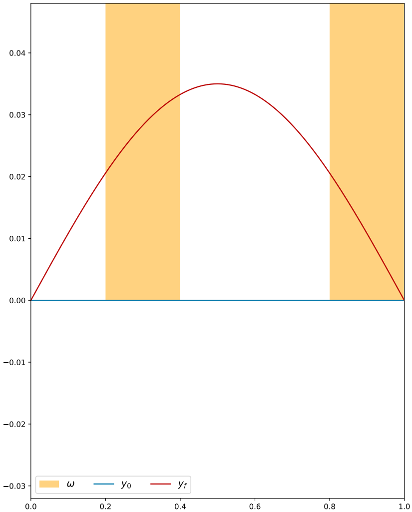

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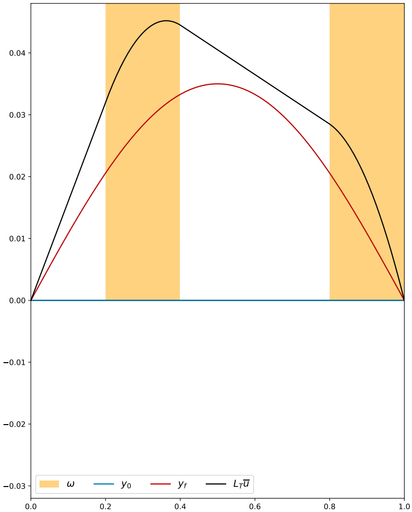
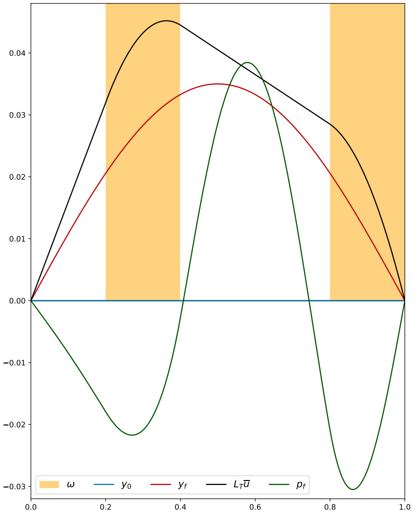


Non-reachable states for the heat equation

$\forall t, x \in [0, T] \times [0, 1]$

$$\begin{cases} \dot{y}(t,x) = \Delta y(t,x) + \mathbb{1}_\omega u(t,x), \\ y(0,x) = y_0(x) = 0, \\ y(t,0) = y(t,1) = 0, \\ 0 \leq u(t,x) \leq 1, \\ y_f(x) = 0.035 \sin(\pi x). \end{cases}$$



Proposition (IH et al., 2025)

y_f is not $\mathcal{U}$-reachable from y_0 in time $T = 1$. Indeed,

$$J(p_f) \in [-0.00072, -0.00011] < 0.$$

Non-reachable states for the heat equation

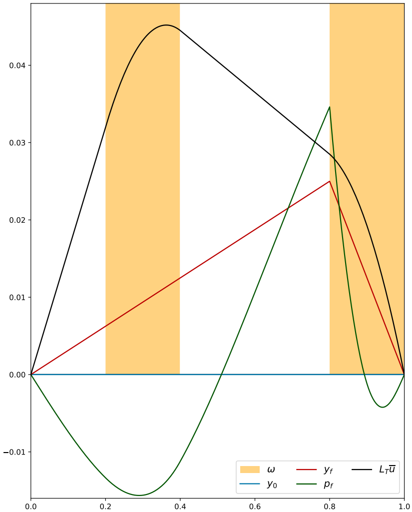
$\forall t, x \in [0,T] \times [0,1]$

$$\begin{cases} \dot{y}(t,x) = \Delta y(t,x) + \mathbb{1}_\omega u(t,x), \\ y(0,x) = y_0(x) = 0, \\ y(t,0) = y(t,1) = 0, \\ 0 \leq u(t,x) \leq 1, \\ yf(x) = 0.025(1 - |2x-1|). \end{cases}$$

Non-reachable states for the heat equation

$\forall t, x \in [0,T] \times [0,1]$

$$\begin{cases} \dot{y}(t,x) = \Delta y(t,x) + \mathbb{1}_\omega u(t,x), \\ y(0,x) = y_0(x) = 0, \\ y(t,0) = y(t,1) = 0, \\ 0 \leq u(t,x) \leq 1, \\ yf(x) = 0.025(1 - |2x-1|). \end{cases}$$



Non-reachable states for the heat equation

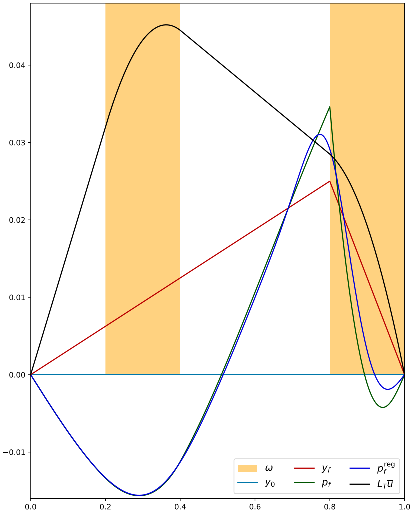
$\forall t,x \in [0,T] \times [0,1]$

$$\begin{cases} \dot{y}(t,x) = \Delta y(t,x) + \mathbb{1}_\omega u(t,x), \\ y(0,x) = y_0(x) = 0, \\ y(t,0) = y(t,1) = 0, \\ 0 \leq u(t,x) \leq 1, \\ yf(x) = 0.025(1 - |2x-1|). \end{cases}$$

Proposition (IH et al., 2025)

y_f n'est pas $\mathcal{U}$-reachable from y_0 in time $T = 1$. Indeed,

$$J(p_f^{reg}) \in [-0.0017, -0.0009] < 0.$$



Non-reachable states for the heat equation

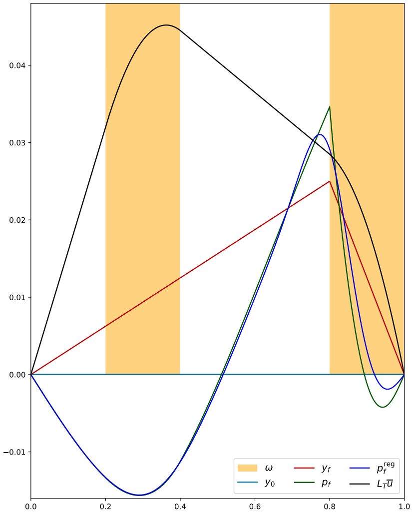
$\forall t, x \in [0,T] \times [0,1]$

$$\begin{cases} \dot{y}(t,x) = \Delta y(t,x) + \mathbb{1}_\omega u(t,x), \\ y(0,x) = y_0(x) = 0, \\ y(t,0) = y(t,1) = 0, \\ 0 \leq u(t,x) \leq 1, \\ yf(x) = 0.025(1 - |2x-1|). \end{cases}$$

Proposition (IH et al., 2025)

y_f n'est pas $\mathcal{U}$-reachable from y_0 in time $T = 1$. Indeed,

$$J(p_f^{reg}) \leq \begin{cases} \text{Eval}(J_d(p_{fh}^{reg})) & = -0.0013 \\ \quad + \ e_r(p_{fh}^{reg}) & \leq \ \ 0.0002 \ < 0. \\ \quad + \ e_d(p_f^{reg}) & \leq \ \ 0.00018 \end{cases}$$



Thank you!

Computer-assisted proofs of non-reachability for linear parabolic PDEs under bounded control constraints, I. H., C. Pouchol, Y. Privat, C. Zhang, 2025, submitted.

Article link

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[CR22] Mo Chen and Lionel Rosier. “Reachable states for the distributed control of the heat equation”. en. In: *Comptes Rendus. Mathématique* 360 (2022). Publisher: Académie des sciences, Paris, pp. 627–639 (cit. on pp. 13, 14).

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